



DAB-8

Seat No. \_\_\_\_\_

**B. Sc. (Sem. II) Examination**

**April – 2022**

**Mathematics**

*(Geometry, Calculus, Matrix Algebra) (New Course)*

Time :  $2\frac{1}{2}$  Hours]

[Total Marks : 70

**Instructions :**

- (1) Attempt any **five** questions.
- (2) All questions carry **equal** marks.

1 (a) Answer the following questions : 4

(1) Write the equation of sphere with center (0, 0, 0) and radius  $a$ .

(2) Find the center of sphere

$$x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0.$$

(3) Define cylinder.

(4) Define right circular cylinder.

(b) Find the co-ordinates of the points where the line 2

$$\frac{x-1}{2} = \frac{y-6}{3} = \frac{z-4}{4} \text{ intersects the sphere } x^2 + y^2 + z^2 = 10.$$

(c) Find the equation of right circular cylinder with axis 3

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \text{ and radius } r.$$

(d) Derive the equation of cylinder of which generator 5

remain parallel to the line  $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$  and passing

through a guiding curve

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0.$$

**2** (a) Answer the following questions : **4**

(1) Write the equation of sphere with center  $(\alpha, \beta, \gamma)$  and radius  $a$ .

(2) Write the vector form of sphere with center  $c(\tau)$  and radius  $a$ .

(3) Find the center of sphere

$$|\vec{r}|^2 + \vec{r} \cdot (-2, 4, -6) + 5 = 0.$$

(4) Write the equation of right circular cylinder whose axis is  $X$  axis with radius  $r$ .

(b) Obtain the condition that the sphere **2**

$x^2 + y^2 + z^2 + 2Ux + 2Vy + 2Wz + d = 0$  touches the plane  $lx + my + nz = P$ .

(c) Find the area of the circle **3**

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0 \text{ and } x - 2y + 2z - 3 = 0.$$

(d) Obtain the equation of tangent plane to the sphere **5**

$$x^2 + y^2 + z^2 + 2Ux + 2Vy + 2Wz + d = 0 \text{ at point } (\alpha, \beta, \gamma).$$

**3** (a) Answer the following questions : **4**

(1) Define rectangular nbhd.

(2) Evaluate  $\lim_{x \rightarrow 1} \left\{ \lim_{y \rightarrow 2} xy^2 \right\}$ .

(3)  $U = y^x$  find  $\frac{\partial U}{\partial x}$ .

(4) State Young's theorem.

- (b) If  $f(x, y) = 0$  then find  $\frac{dy}{dx}$ . 2
- (c) State and prove Euler's theorem for homogeneous function of two variables. 3
- (d)  $U = \sin^{-1}(x - y)$ ,  $x = 3t$ ,  $y = 4t^3$  prove that  $\frac{dU}{dt} = \frac{3}{\sqrt{1-t^2}}$ . 5
- 4 (a) Answer the following questions : 4
- (1) Define circular nbhd.
- (2)  $U = \log(x^2 + y^2)$  find  $\frac{\partial u}{\partial y}$ .
- (3) Evaluate  $\lim_{x \rightarrow 0} \left\{ \lim_{y \rightarrow 0} \frac{xy^2}{x^3 + y^3} \right\}$ .
- (4)  $x = r \cos \theta$  find  $\frac{\partial x}{\partial r}$ .
- (b)  $U = F(y - z, z - x, x - y)$  prove  $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$ . 2
- (c) If  $ax^2 + 2hxy + by^2 = 1$  then prove 3
- $$\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}.$$
- (d)  $U = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ ,  $x \neq y$  prove that 5
- (1)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
- (2)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u.$

- 5** (a) Answer the following questions : **4**
- (1) Define Jacobian.
  - (2) Define Extreme value.
  - (3) Define Critical points.
  - (4) Define Local maxima and minima.
- (b)  $F(x, y, z) = \sqrt{x^2 + y^2 + z^2}$  then find approximate **2**  
value of  $f(1.9, 2.01, 4.8)$ .
- (c) Explain  $\log xy$  in power of  $x-1$  and  $y-1$ . **3**
- (d)  $U = \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$  then the extreme value of U when **5**

$$lx + my + nz = 0 \text{ and } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ can be obtained}$$

$$\text{by equation } \frac{l^2 a^4}{1 - a^2 U} + \frac{m^2 b^4}{1 - b^2 U} + \frac{n^2 c^4}{1 - c^2 U} = 0.$$

- 6** (a) Answer the following questions : **4**
- (1) Define global maxima and minima.
  - (2)  $\frac{\partial(U, V)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(U, V)} =$
  - (3) Define extreme point.
  - (4)  $x = r \cos \theta, y = r \sin \theta$  find  $\frac{\partial(x, y)}{\partial(r, \theta)}$ .
- (b) Find maxima and minima for **2**

$$f(x, y) = xy + 27 \left( \frac{1}{x} + \frac{1}{y} \right).$$

(c) Expand  $x^2y + 3y - 2$  in powers of  $x - 2$  and  $y - 3$ . 3

(d)  $U = x^2 + y^2 + z^2$ , 5

$$V = xy + yz + zx,$$

$$W = x^3 + y^3 + z^3 - 3xyz$$

prove that  $\frac{\partial(U, V, W)}{\partial(x, y, z)} = 0$ .

7 (a) Attempt the following questions : 4

(1) Define Symmetric matrix.

(2) Define Hermitian matrix.

(3) Define Idempotent matrix.

(4) Find the trace of matrix

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

(b) For non singular matrix  $A$  and  $B$  prove that 2

$$(AB)^{-1} = B^{-1}A^{-1}.$$

(c)  $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$  is orthogonal matrix then find  $l, m, n$ . 3

(d) Find  $A^{-1}$  by using preliminary row operations where 5

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 7 \\ 3 & 4 & 4 \end{bmatrix}.$$

8 (a) Attempt the following questions : 4

(1) Define skew symmetric matrix.

(2) Define orthogonal matrix.

(3) Define skew hermitian matrix.

(4) Give an example of  $3 \times 3$  lower triangular matrix.

- (b) Find rank of  $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$ . 2
- (c)  $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 0 & 5 \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$  prove  $(AB)^T = B^T A^T$ . 3
- (d) Prove every square matrix can be uniquely expressed as the sum of symmetric matrix and skew-symmetric matrix. 5
- 9** (a) Answer the following questions : 4
- (1) Define eigen value.
  - (2) Define characteristics equation.
  - (3) Define consistent system of equation.
  - (4) Find characteristics equation of  $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ .
- (b) Find eigen value and eigen vector for  $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ . 2
- (c) Show the equations : 3
- $$x + y + z = -3$$
- $$3x + y - 2z = -2$$
- $$2x + 4y + 7z = 7 \quad \text{are not consistent}$$
- (d) State and prove Cayley Hamilton theorem. 5
- 10** (a) Answer the following questions : 4
- (1) Define eigen vector.
  - (2) Define augmented matrix.
  - (3) Define inconsistent system of equation.
  - (4) Find eigen value of  $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ .

(b) Matrix  $A - \lambda I$  is singular  $\Leftrightarrow \lambda$  is an eigen value of matrix  $A$ . **2**

(c) Solve : **3**

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x + y - z = 1$$

(d) Verify Cayley Hamilton theorem for  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ . **5**

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